SAN ANTONIO
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Creating Models of Truss Structures with Optimization

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Truss Structures

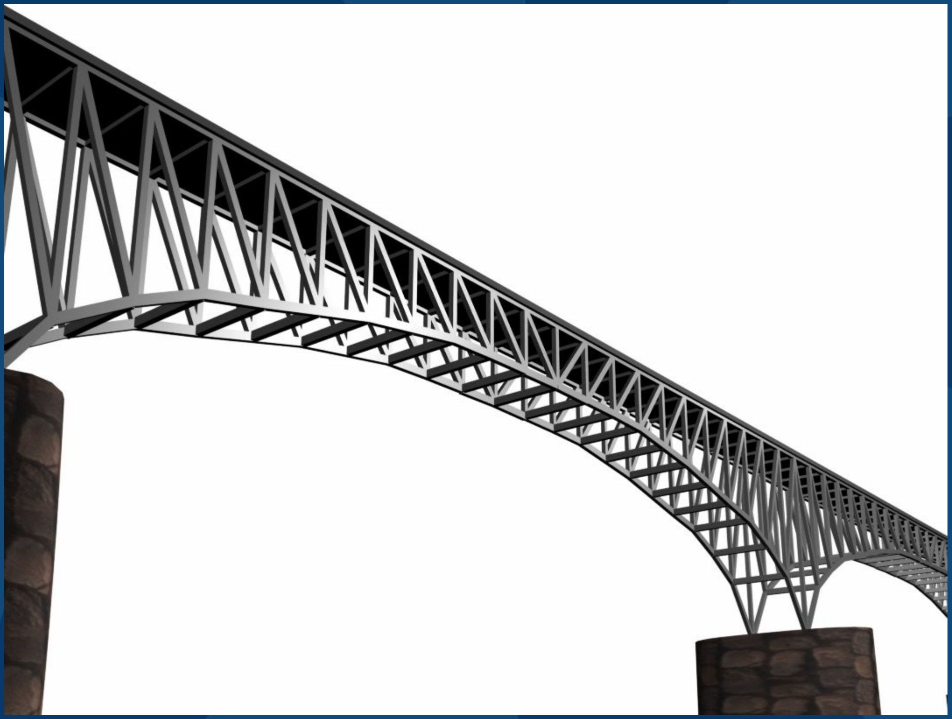
- **Rigid beams**
  - Axial forces only
- **Pin-connected**
  - Concentric joints
  - Welded or bolted
- **Bridges, towers, exoskeletons**

Why do we want a way to generate truss structures?

- **Common**
- **Complex**
  - Many joints and beams
- **Time-consuming to build by hand**

Our Approach

- Use optimization to design truss structures to support user-specified loads



7 minutes, 275Mhz R10000 SGI Octane

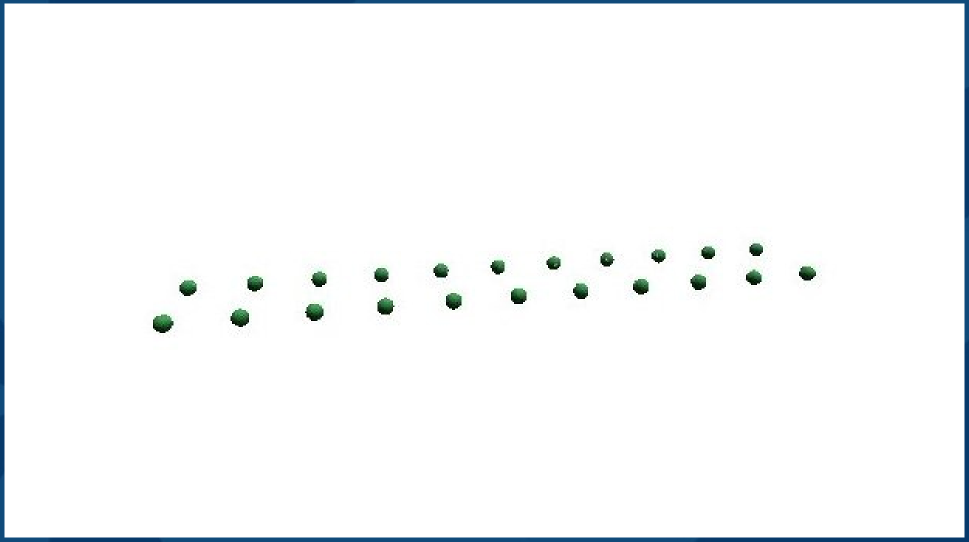
How Do We Model Truss Structures?

- **Mass is “lumped” at pin-joints**
  - Structure much larger than beams
- **Discrete external loads**
  - Road surface, cars, utility wires, etc.
- **Anchored to ground**

How Does it Work?

User specifies:

- Load locations



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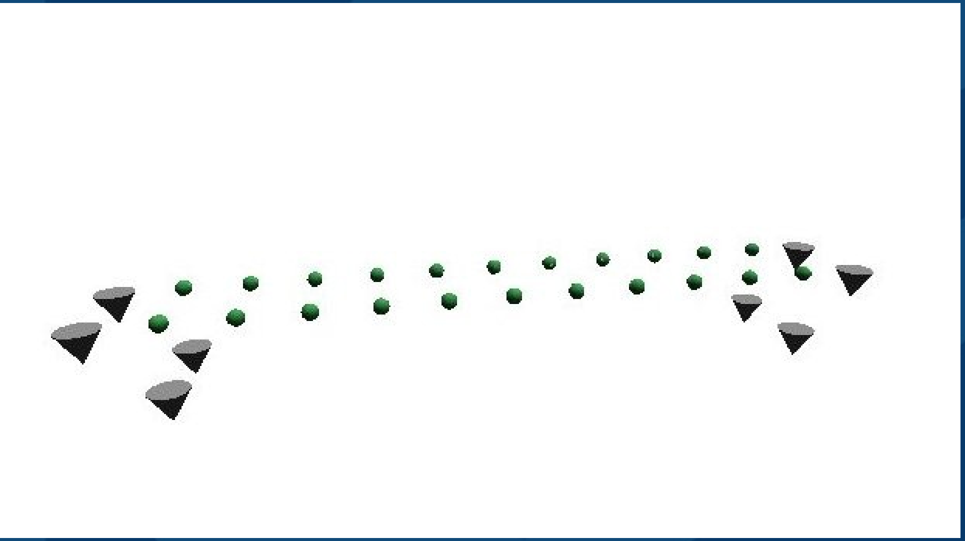

How Does it Work?

User specifies:

- Load locations
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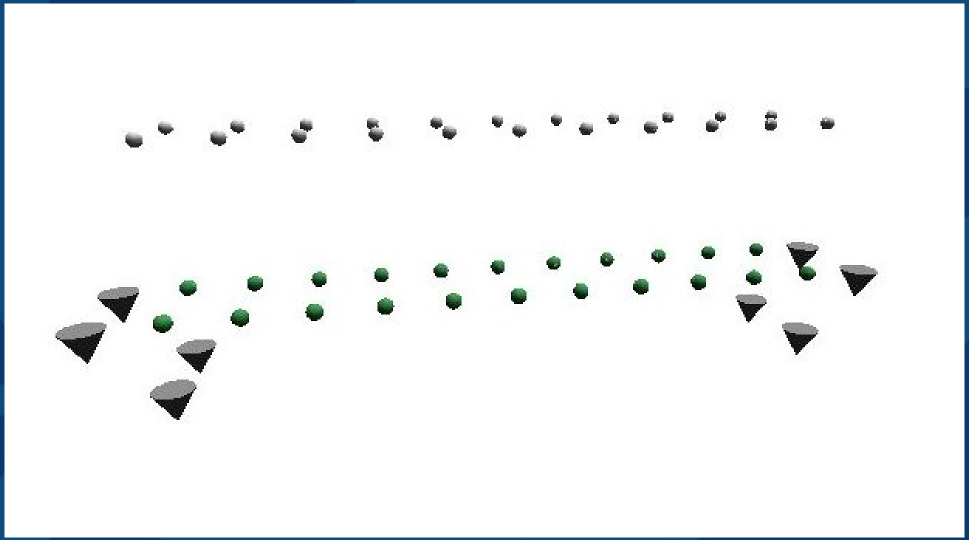
Code adds:

- Pin-joints



How Does it Work?

User specifies:

- Load locations
- Anchor locations

Code adds:

- Pin-joints
- Beams connecting joints, anchors, and loads

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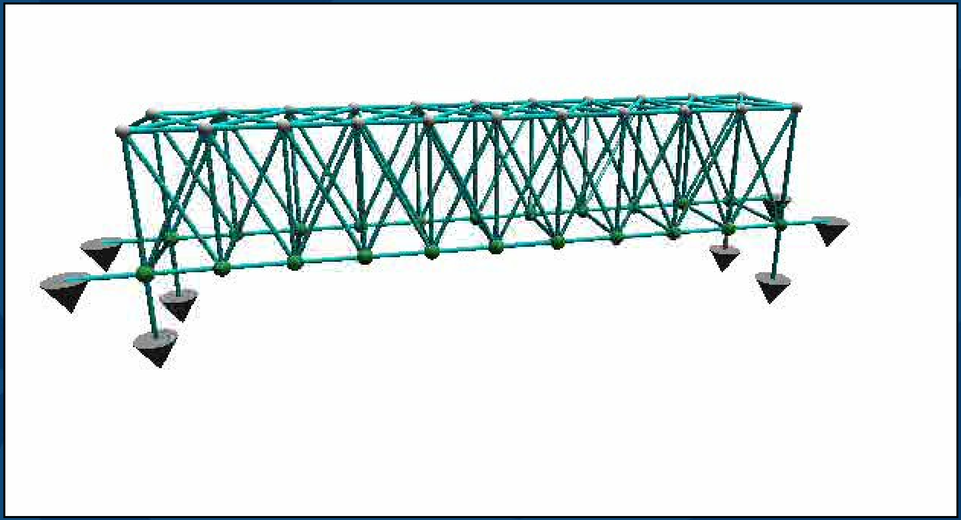


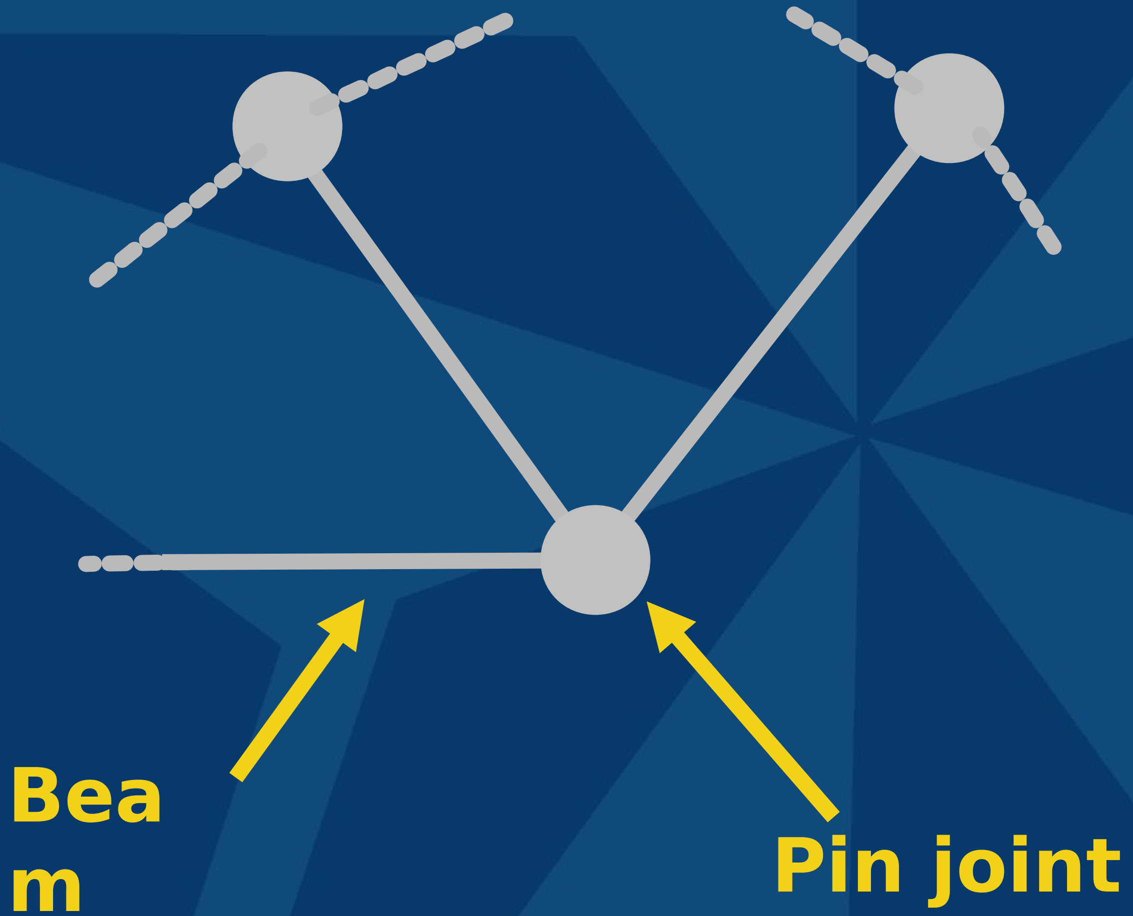
2 minutes, 275Mhz R10000 Octane

Optimize to find best structure

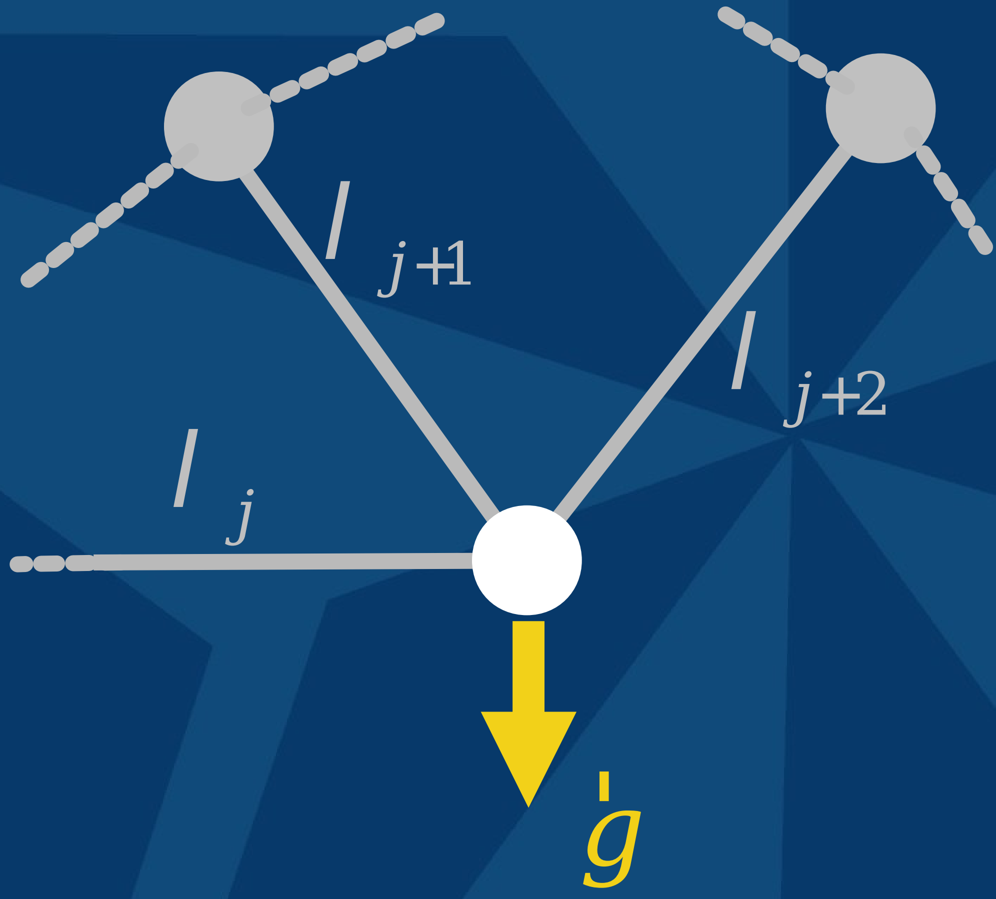
Why Use Optimization?

- **Truss designs are usually not dominated by aesthetic concerns**
  - Utilitarian
  - Inexpensive (minimal mass)
- **Beam and joint construction**
  - Simple mathematical representation

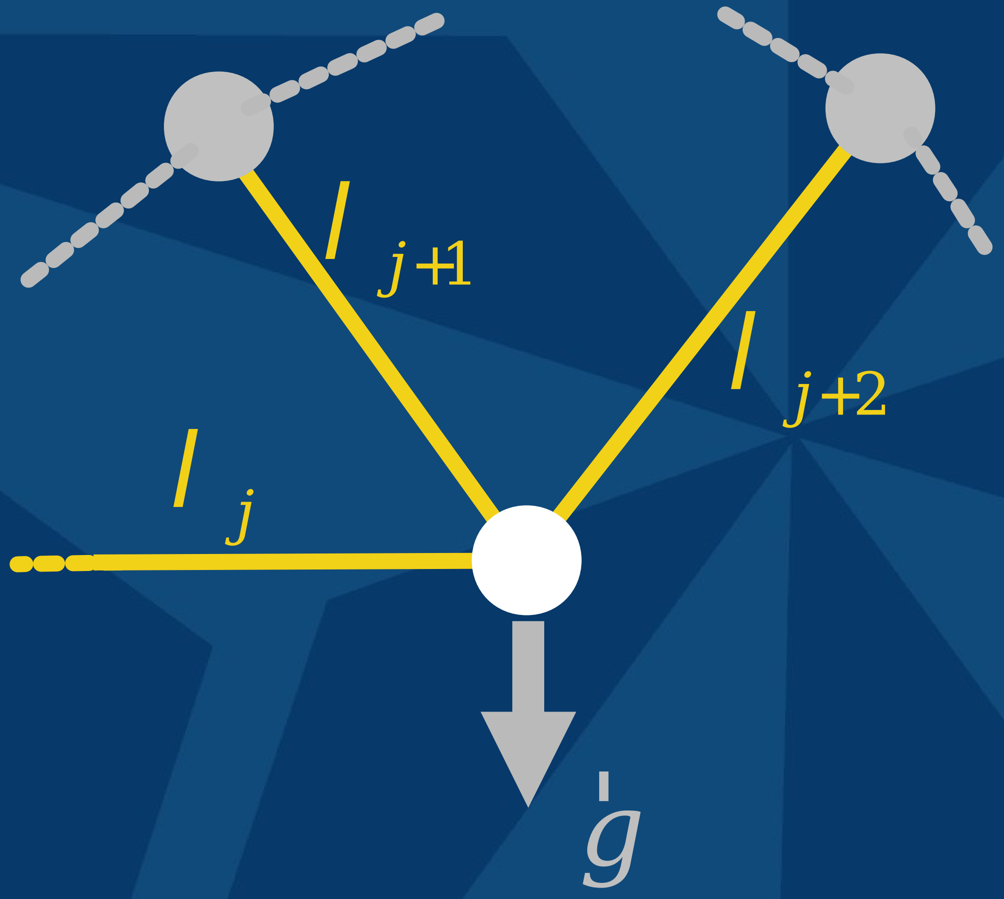
Joints and Beams



Joints and Beams



Joints and Beams



Forces on a Pin-Joint

$$\vec{F}_i = \vec{g} m_i + \sum_{j=1}^{B_i} \frac{\vec{l}_j}{\left\| \vec{l}_j \right\|} \lambda_j$$

For stability:

$$\vec{F}_i = 0$$

$\vec{F}_i$ - Force on joint *i*

m_i - Mass of joint *i*

$\vec{l}_j$ - Vector of beam *j*

λ_j - Force of beam *j*

Mass Functions

A joint's mass depends on:

- External loads
- The beams that connect to it

A beam's mass depends on:

- Length $\left\| \vec{l}_j \right\|$
- Workless force it exerts λ_j
- Tension or compression

Beam Mass Functions

Under tension:

- $\lambda_j < 0$

$$m_j = -k_T \lambda_j \left\| \vec{l}_j \right\|$$

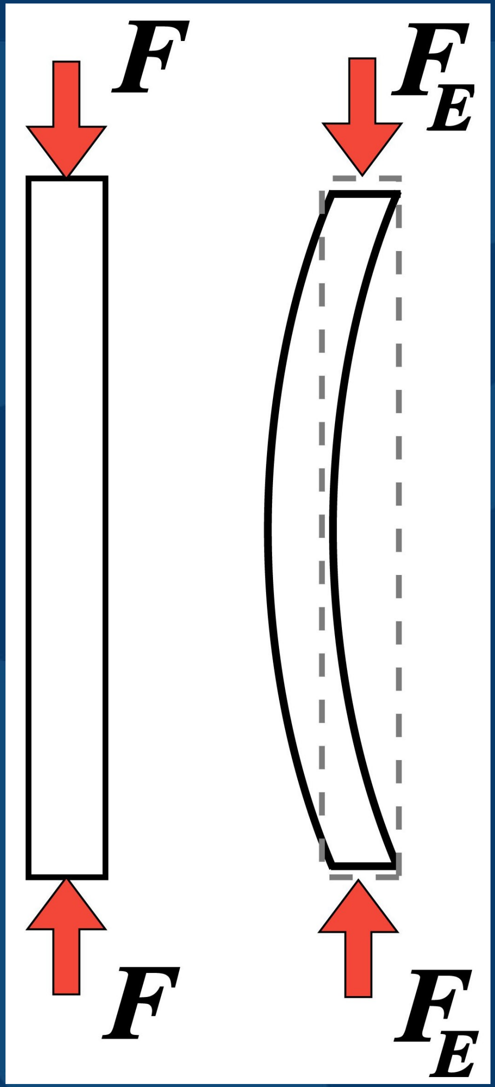
- **Length**
- **Area**
  - Proportional to workless force

Beam Mass Functions

Under compression:

- $l_j > 0$

- **Euler buckling**
  - Length
  - Area
  - “Radius of gyration”



Euler Buckling

Force limit: $\lambda_{max} = \dfrac{\pi^2 E r^2 A}{\left\| \vec{l}_j \right\|^2}$

$$A^2 \propto \frac{\lambda_{max} \left\| \vec{l}_j \right\|^2}{\pi^2 E}$$

$$m_j = \rho A_j \left\| \vec{l}_j \right\| = k_C \sqrt{\lambda_j} \left\| \vec{l}_j \right\|^2$$

The Optimization Problem

$$\min G(\vec{l}, \vec{p}) = \sum_{i=1}^{N_J} m_i$$

- **Minimize mass**

The Optimization Problem

$$\min G(\vec{l}, \vec{p}) = \sum_{i=1}^{N_J} m_i$$

$$\text{s.t.} \quad \vec{F}_i(\vec{l}, \vec{p}) = 0 \qquad i = 1 \ldots N_J$$

- **Subject to force balance constraints**

The Optimization Problem

$$\min G(\vec{\lambda}, \vec{p}) = \sum_{i=1}^{N_J} m_i$$

$$\text{s.t.} \quad \vec{F}_i(\vec{\lambda}, \vec{p}) = 0 \qquad i = 1 \ldots N_J$$

$$l_{\min} \le \left\| \vec{l}_j \right\| \le l_{\max} \qquad j = 1 \ldots N_B$$

$$\lambda_i \le \lambda_{\max} \qquad i = 1 \ldots N_J$$

- **Subject to “realism” constraints**

The Optimization Problem

$$\min G(\vec{l}, \vec{p}) = \sum_{i=1}^{N_J} m_i$$

$$\text{s.t.} \quad \vec{F}_i(\vec{l}, \vec{p}) = 0 \qquad i = 1 \ldots N_J$$

$$l_{\min} \le \left\| \vec{l}_j \right\| \le l_{\max} \qquad j = 1 \ldots N_B$$

$$l_i \le l_{\max} \qquad i = 1 \ldots N_J$$

Optimize with respect to:

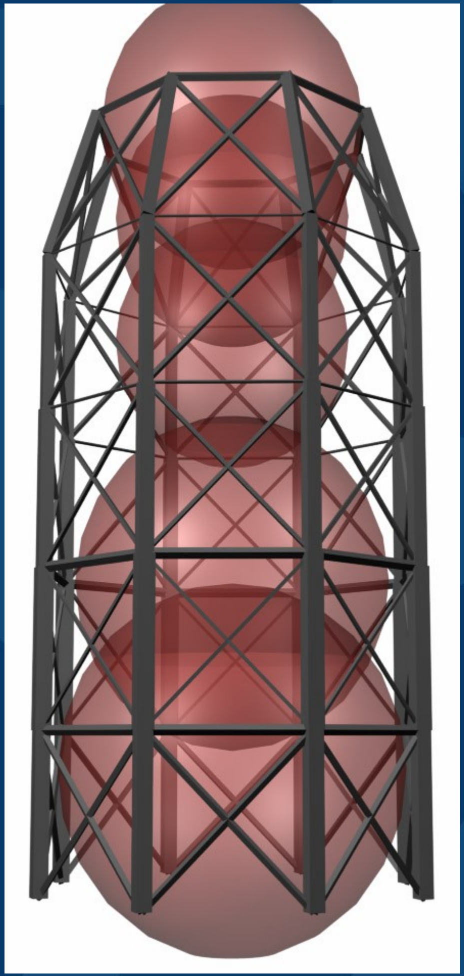
- **Workless forces:** $\vec{l}$
- **Positions of pin-joints:** $\vec{p}$

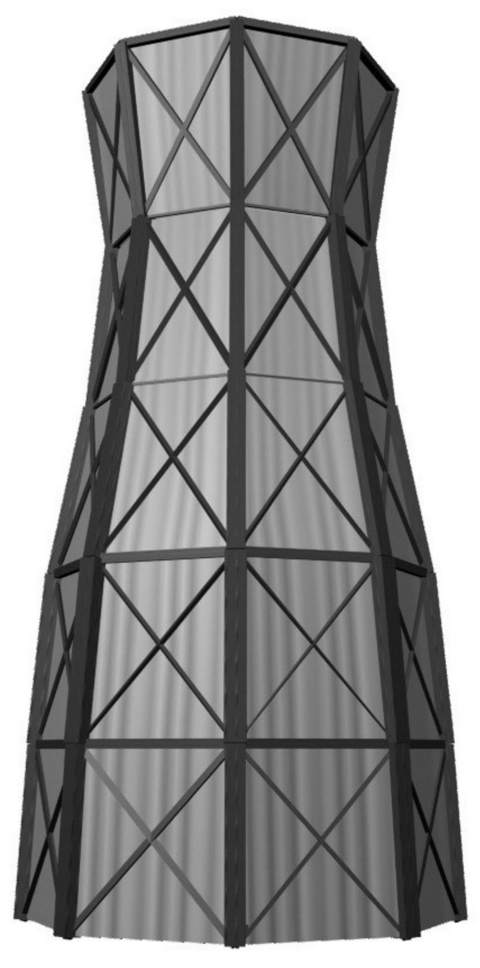
Optimization Method

Sequential Quadratic Programming:

- **Fast and robust**
- **Handles:**
  - Non-linear objective function
  - Non-linear constraints
- **Local minima**

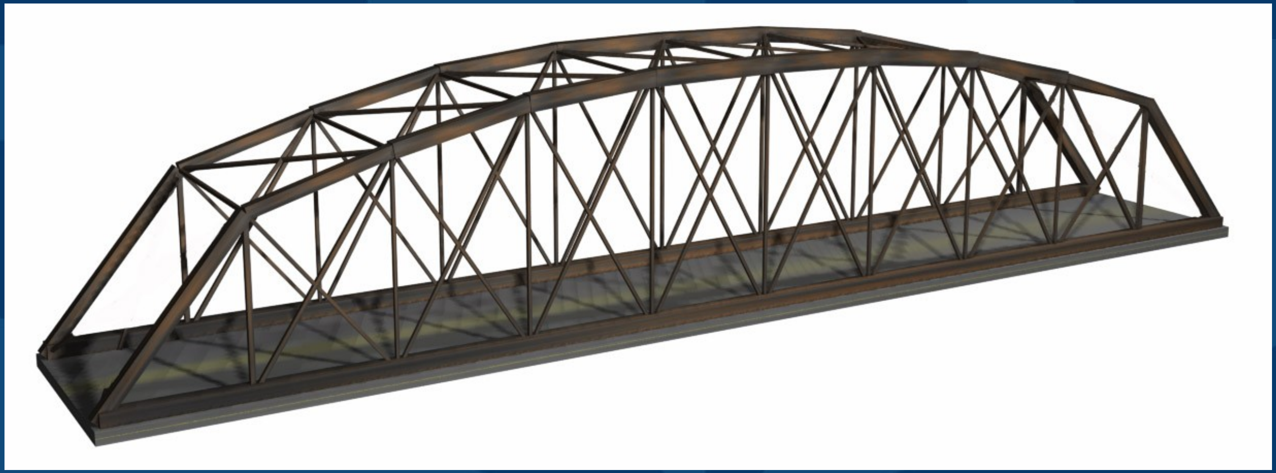
Spherical Obstacle Constraints



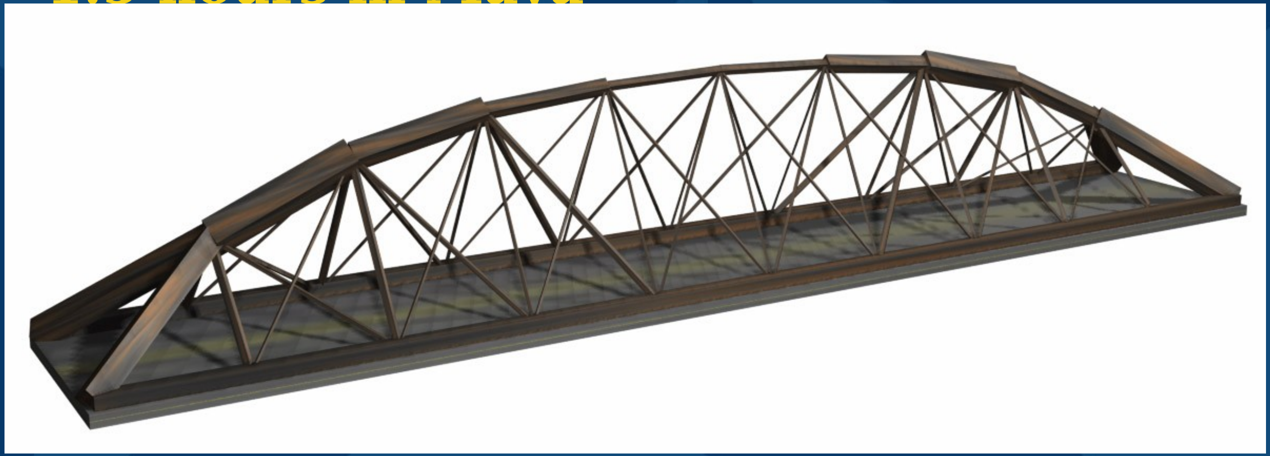


5 minutes, 275Mhz R10000 SGI Octane ~1 hour in Maya

Planar Constraints

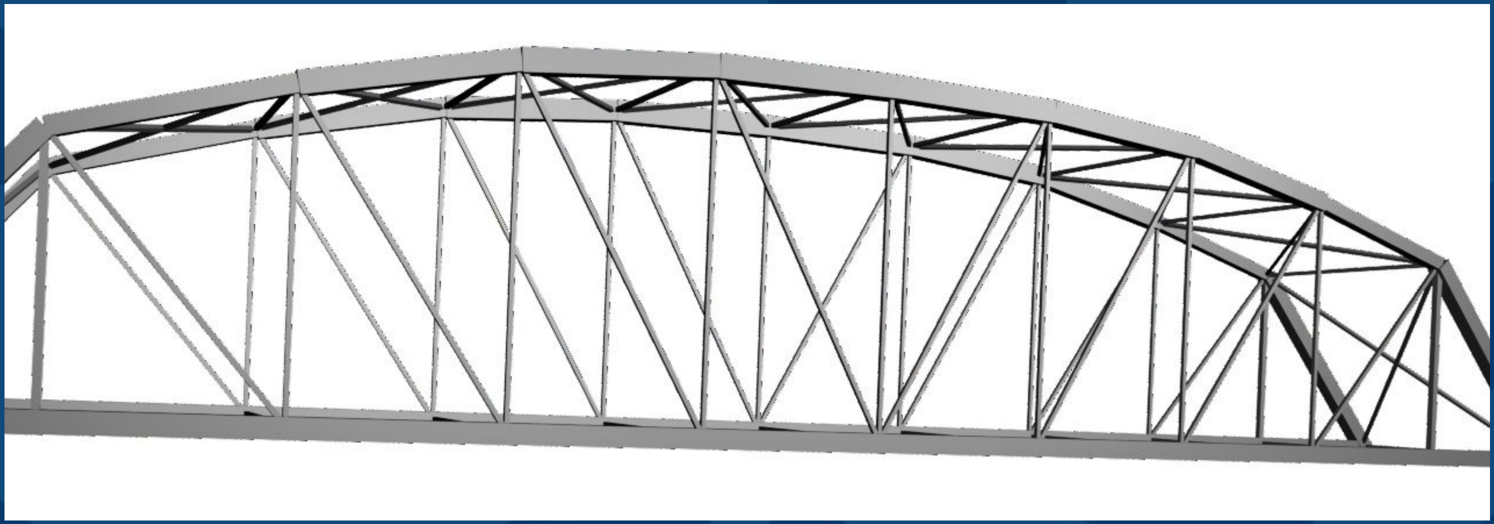


3 minutes, 275Mhz R10000 Octane
~1.5 hours in Maya



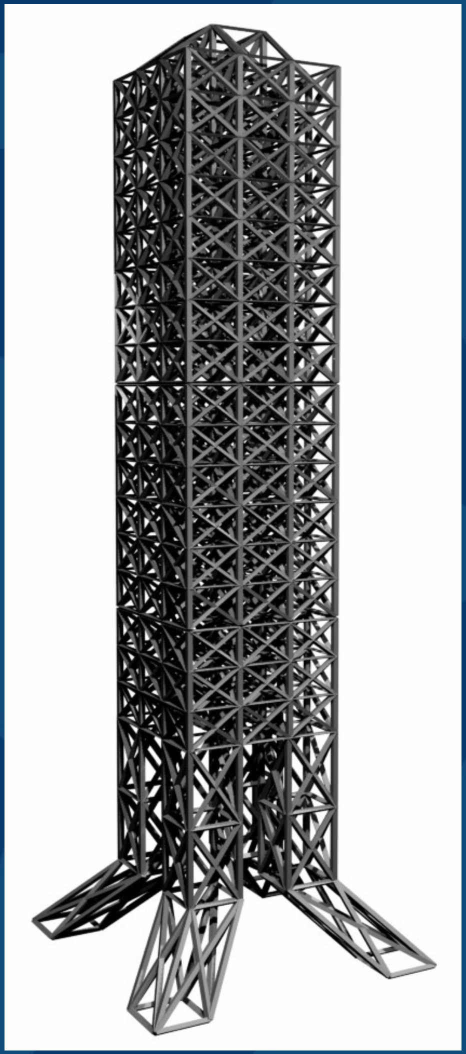
3 minutes, 275Mhz R10000

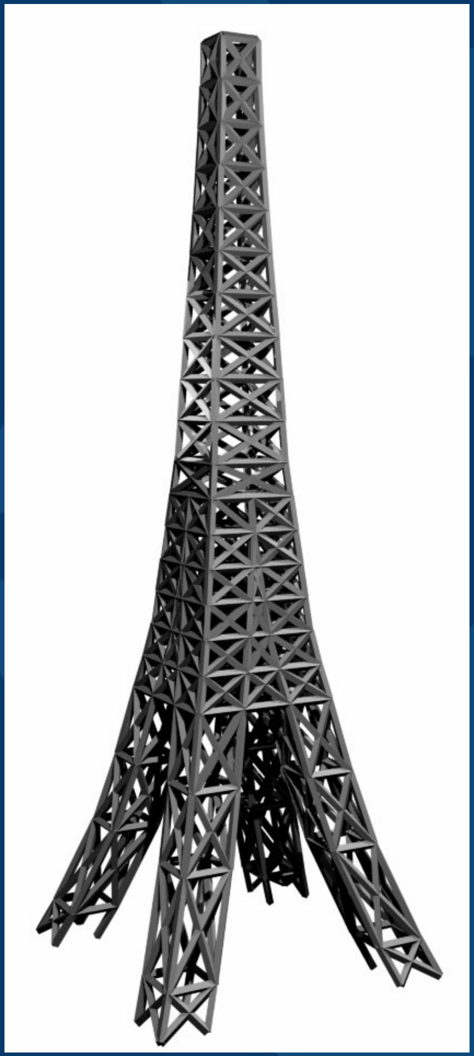
Railroad Bridge



3 minutes, 275Mhz R10000 Octane

Eiffel Tower





15 minutes, 275Mhz

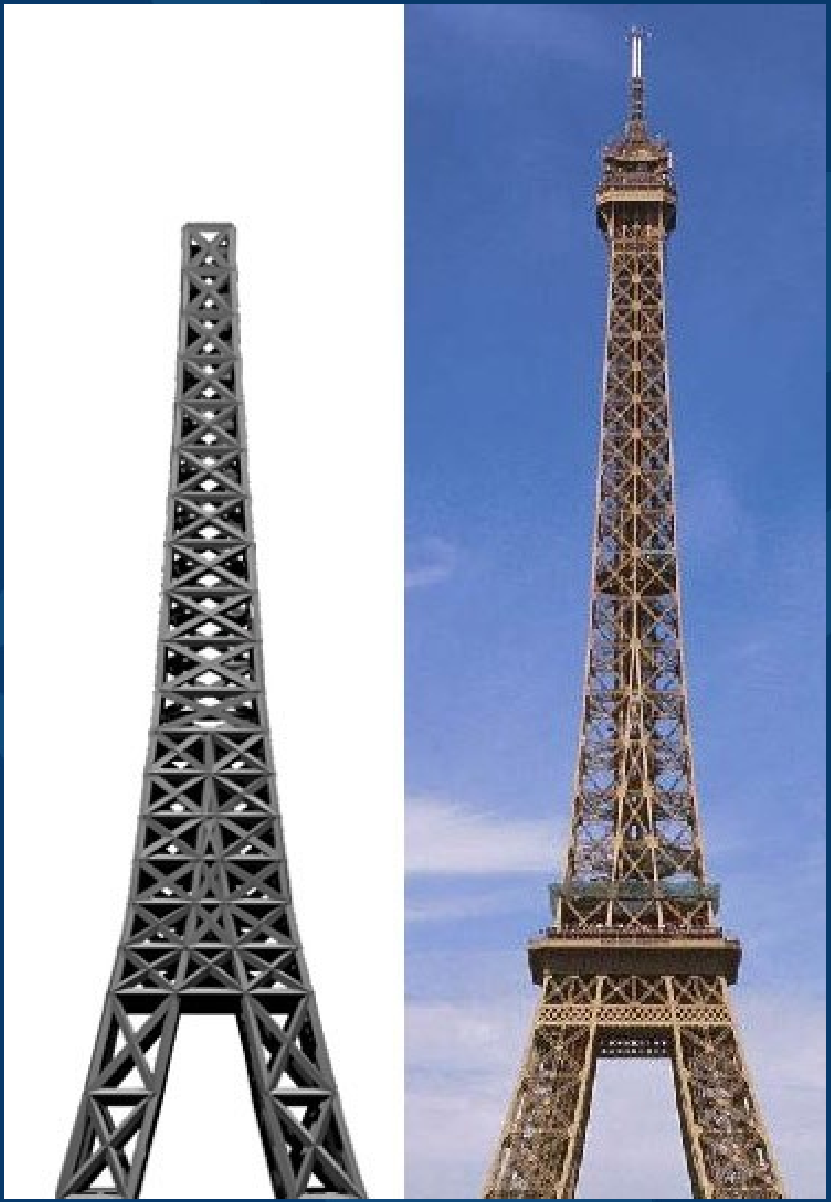
Limitations and Future Work

- **Simple Objective Function**
  - True cost of construction
- **Simple mass functions**
  - Better column formula
- **Single set of loads**
  - Multi-objective optimization

Future Work

Aesthetic criteria

- Symmetry
- Visual weight
- Geometric forms



15 minutes, 275Mhz